A Union of Professionals
Common Core
State Standards

Dear Parents and Caregivers,

This is another letter about the expectations of the new Common Core State Standards for Mathematics. We continue to work to prepare your child to be able to meet the demands of college and/or the work place in the 21st century. This letter explains how students can relate the mathematical practices to one of the major ideas of geometry in eighth grade, the Pythagorean Theorem. It also shows where we may encounter this mathematics in real life.

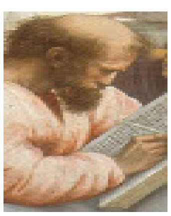
8.G.6 Understand and apply the Pythagorean Theorem

- Explain a proof of the Pythagorean Theorem and its converse.
- Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
- Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Understanding and applying the Pythagorean Theorem

In order to understand and prove this theorem most students will make sense by using a diagram, attaching numbers to it, and determining whether the result is what the theorem says. The numbers and diagram will help them build a valid argument or proof. Here is what they will learn.

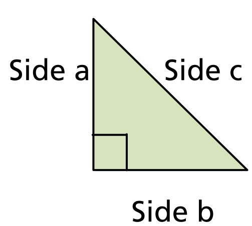
Pythagoras' Theorem

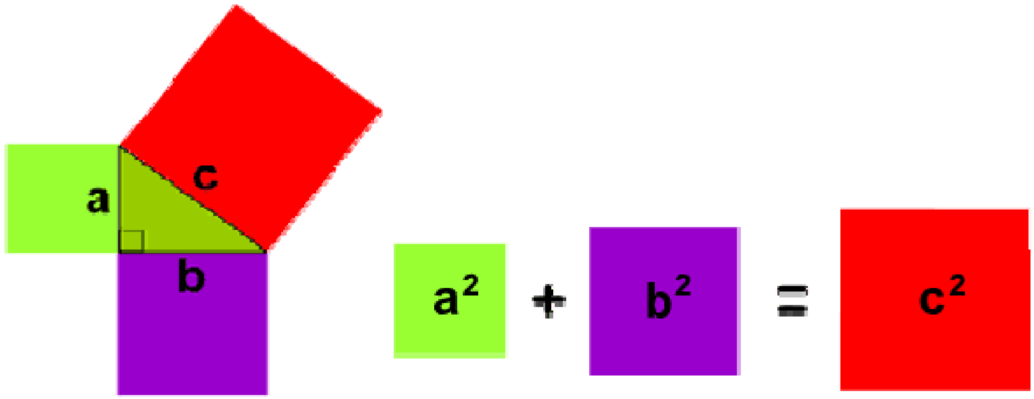


Years ago, a man named Pythagoras found an amazing fact about triangles:
If the triangle has a right angle (90°) ...
... and you make a square on each of the three sides—a, b, and c,—then ...
... the square built on the longest side has the exact same area as the other two squares' areas added together!

It is called "Pythagoras' Theorem" and can be written in one short equation:

$$a^2 + b^2 = c^2$$





Note:

- c is the longest side of the triangle
- a and b are the other two sides and they form the right angle

Modeling the theorem

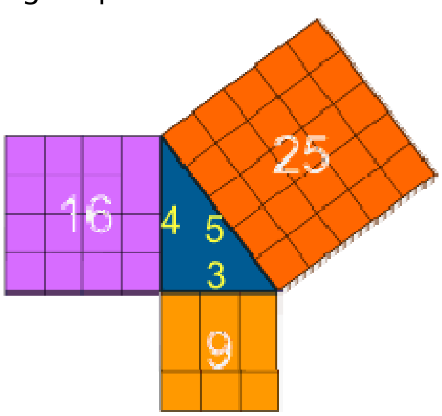
The illustration above is a model of the theorem. It shows what the words mean. Using models is a mathematical practice. The longest side of the triangle is called the "hypotenuse", so the formal theorem states:

In a right-angled triangle
the square of the hypotenuse is equal to
the sum of the squares of the other two sides.

Proving the areas are the same

Another of the mathematical practices is for students to build valid mathematical arguments. Let's see if the theorem really works using an example.

Example: A triangle with side lengths of 3 cm, 4 cm, and 5 cm, called a "3,4,5" triangle, has a right angle. To check whether the areas are the same, we have shown the areas of each square in square centimeters. Do the areas of the two smaller squares equal that of the larger square?



$3^2 + 4^2 = 5^2$

(3*3) + (4*4) = (5*5)

We calculate:

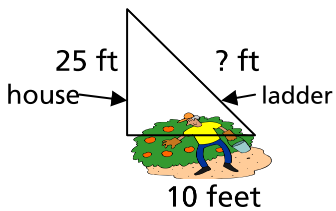
9 + 16 = 25

It works!

After working with models, students can visualize the squares on the sides, they have a picture of what it means to "square" a number. That is, 4^2 means you are making a 4 by 4 square with 4 units on each side. So students can work without actually drawing the squares on the side and find 4^2 by simply multiplying 4 x 4.

Why Is this theorem important and useful?

This mathematical theorem has uses in real life. Suppose you are locked out of your house and know there is an unlocked window on the second floor 25 feet above the ground. If you can borrow a ladder to reach the window you can get in. There are bushes that mean the legs of the ladder must be placed 10 feet from the side of the house. How long must the ladder be to reach the window? You can figure this out by using 25 ft and 10 feet as two sides of a triangle that form a right angle with the house.



$10^2 + 25^2 = ?^2$

$(10*10) + (25*25) = ?^2$

$100 + 625 = ?^2$

$725 = ?^2$

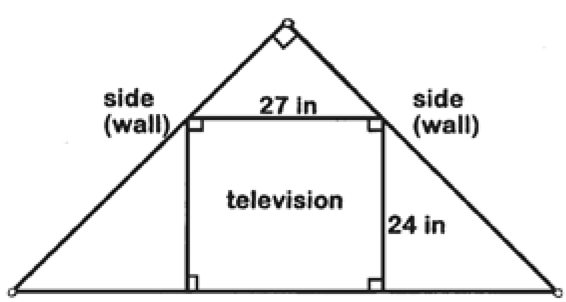
$\sqrt{725} = 26.9258...$ so the ladder should be at least 27 feet long.

Family support: Ask your child to help you design a new corner cabinet for the TV, games and DVDs. The key information is the dimensions of the TV so you know how deep to make the shelf. We also want the shelf to be the same length on each side (along the two walls).

Corner Cabinet

(Based on the Entertainment Center task at www.exemplars.com/education-materials/free-samples)

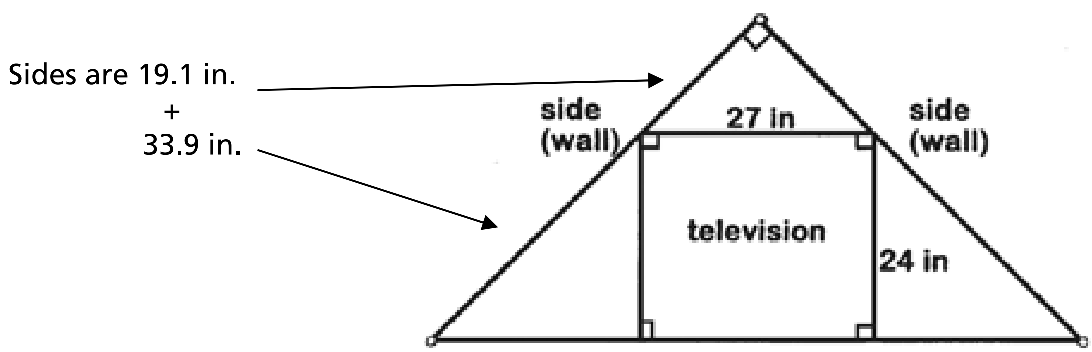
In designing a new corner cabinet for our TV, I had to figure out how deep to make it so that the TV we currently have would fit. The cabinet must be the same length on each side along the walls that meet in the corner. This is an overhead view. Give the larger diagram that is attached to your child and see if he/she can solve it.



For problems like this, students must show all their work and explain what the symbols represent and how they reach each conclusion. There are different ways to solve the problem. Here is one, with a step by step explanation. If your child has difficulty, have him or her connect the explanation to the diagrams and then practice explaining.

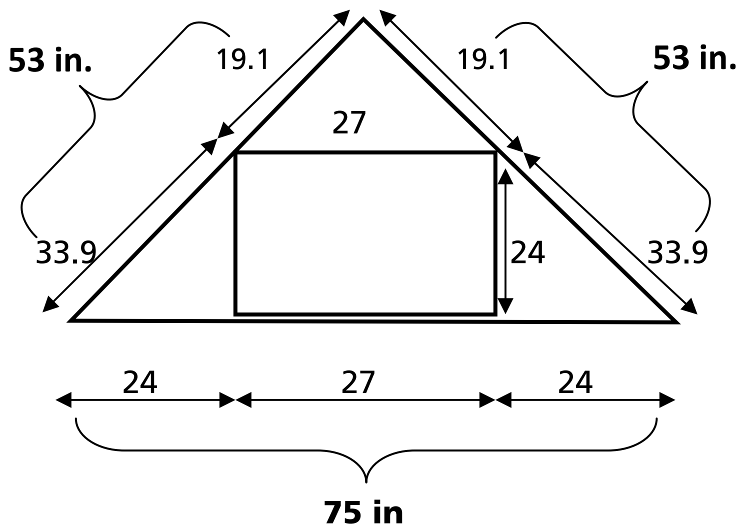
One corner cabinet solution

1. We know the large triangle is an isosceles triangle because we specified that the sides had to be the same length.
2. Since the top angle is 90° and the bottom ones must be equal, the bottom angles are each 45°*.*
3. Since the two bottom triangles each have a 90° angle and a 45° angle the third angle of each must also be 45° since every triangle has a total of 180° for all three angles.
4. A student may identify the small triangle at the top as another isosceles triangle and reason that the two bottom angles are each 45°*;* or they may reason that the straight angle that includes the top of the lower triangles (45°), the corner of the TV set (90°) and the bottom of the top triangle require those angles to be 45°*.*
5. They can use the Pythagorean theorem to find the length of the sides of the top small triangle. The square of the hypotenuse (27 x 27) is 729 in. Thus the squares of the other sides together are 729 in. Divided equally for the other two sides, each is 364.5 in^2, so each side is $\sqrt{364.5}$ or19.1 in. (Use a calculator to find this.) Thus, 19.1 in. is the length of the sides of the top triangle.
6. Similar calculations are done for the lower small triangles to figure out the length of *their hypotenuses*. The lower triangles have the same angle measures as the one at the top (90° and two 45° angles, so we know it is isosceles, too*.)* One side is given as 24 in. so we square that to find the measure of the two sides squared: 24 x 24 = 576. Adding the areas of the two squares gives us 1152 in^2 for the area of the non-hypotenuse legs. Therefore, the length of the hypotenuse is $\sqrt{1152}$ = 33.9 in.



7. Each side of the cabinet will be 19.1 + 33.9 in or 53 inches long.
8. Now we must find the measure of the front of the cabinet, the bottom of the large triangle. We found that the legs of the lower triangles are 24 in and we know the middle part of that line is part of a rectangle with its opposite side

being 27 in, so it is also 27 in. What remains is to add the partial measurements. 24 + 27 + 24 = 75 in for the front of the cabinet! The triangular top measures 53 by 53 by 75 inches (approximately).



If we know the lengths of **two sides** of a right-angled triangle, we can find the length of the **third side**. (But remember it only works on right-angled triangles!) This comes in handy for real world situations.

The National PTA has created resources to help parents support their children. Information can be found at www.pta.org/common_core_state_standards.asp.

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Grade 8 teacher

Student Practice
Entertainment Center

How long must the sides and front of the entertainment cabinet be to accommodate this television set? Show your work and explain how you reach your conclusions.